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Roberto J. Acosta
Lewis Research Center
Cleveland, Ohio

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COMPUTATION OF THE RADIATION CHARACTERISTICS OF A GENERALIZED PHASED ARRAY

Roberto J. Acosta
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

With the advent of monolithic microwave integrated circuit (MMIC) technology, the phased array has become a key component in the design of advanced antenna systems. Array-fed antennas are used extensively in today's multiple beam satellite antennas. In this report, a computer program based on a very efficient numerical technique for calculating the radiated power (Romberg integration), directivity and radiation pattern of a phased array is described. The formulation developed is very general, and takes into account arbitrary element polarization, E and H-plane element pattern, element location, and complex element excitation. For comparison purposes sample cases have been presented. Excellent agreement has been obtained for all cases. Also included in appendixes A and B are a user guide and a copy of the computer program.

INTRODUCTION

One of the most important radiation characteristics of an antenna system is the directivity. Accurate determination of this parameter is essential for the analysis and design of advanced antenna systems. In general, for most commonly used array elements such as open-ended waveguides, horns, or micro-strip patch antennas, an analytical expression of the $(\cos \theta)^q$ -type can be used to properly tailor actual element patterns (ref. 1). Experimentally measured element patterns could include mutual coupling, which may be significant in large arrays. Expressions for the directivity and array radiation pattern of a single element and a rectangular array using $(\cos \theta)^q$ -type element patterns have been reported (refs. 2 to 11).

It is the purpose of this work to generalize these results and obtain an efficient numerical technique for computing the directivity and the antenna radiation pattern of the generalized array. The generalized array characteristics used in this report includes arbitrary element location, element pattern ($(\cos \theta)^q$ -type, other analytically describable functions or experimentally measured), element polarization and element excitation.

ARRAY RADIATION PATTERN

The geometry of the generalized array is shown in figure 1. Given the array geometry and element characteristics, the generalized array problem can be defined as: (1) to determine the power radiated and directivity at a given observation point (this is usually taken in the far-field zone), (2) to determine the co-polarization and cross-polarization component of the electric field (using Ludwig's criterion (ref. 12)). In solving this problem, two sets of coordinate system are used. Figure 2 shows a typical element coordinate system and the reference coordinate system, with the z-axis in the same

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direction. For an array of M elements located arbitrarily in the reference coordinate system (fig. 1), the m^{th} element radiated field is given by equation A1.1:

$$\vec{E}_m(\vec{r}_m') = I_m \left[\frac{e^{-jk r_m'}}{r_m'} \hat{\theta}' U_{Em}(\theta') \left(a_m e^{j\psi_m} \cos \varphi' + b_m \sin \varphi' \right) + \hat{\varphi}' U_{Hm}(\theta') \left(-a_m e^{-j\psi_m} \sin \varphi' + b_m \cos \varphi' \right) \right] \quad (\text{A1.1})$$

for $0 \leq \theta' \leq \pi/2$, where

I_m M^{th} element complex excitation coefficient
 U_{Em}, U_{Hm} M^{th} element E and H plane pattern
 a_m, b_m, ψ_m M^{th} element polarization parameters (see table I)
 K wave number $2\pi/\lambda$
 r_m', θ', φ' spherical coordinates in the element coordinate system

The element pattern U_{Em}, U_{Hm} can be described with an analytical expression ($(\cos \theta)^q$ -type or other functions) or with experimentally measured data (discrete). If measured data are used, the pattern may include mutual coupling effects. The polarization parameters in table I are subject to the normalization describe by

$$a_m + b_m = 1 \quad (\text{A1.2})$$

The electric field described by equation (A1.1) is in the element coordinate system. The total electric field due to all M elements is the superposition of the electric field of each element of the array. The total electric field is given by

$$\vec{E}(\vec{r}) = \sum_{m=1}^M \vec{E}_m(\vec{r}) \quad (\text{A1.3})$$

where the vector fields $\vec{E}(\vec{r})$ and $\vec{E}_m(\vec{r})$ are defined in the reference coordinate system. A transformation of $\vec{E}_m(\vec{r}_m')$ (eq. (A1.1)) in the element coordinate system into $\vec{E}_m(\vec{r})$ in the reference coordinate system is described next. Figure 3 shows a detailed description of these coordinate systems. The transformation of coordinates for this problem only involves a translation. The transformation procedure is outlined as follows. Knowing the observation coordinates (r, θ, φ) and m^{th} element location (x_m, y_m, z_m) , the observation point in the primed coordinate system is found by using:

$$\begin{aligned} x &= r \sin \theta \cos \varphi \\ y &= r \sin \theta \sin \varphi \\ z &= r \cos \theta \end{aligned} \quad (\text{A1.4a})$$

$$\begin{aligned} x' &= x - y_m \\ y' &= y - y_m \\ z' &= z - z_m \end{aligned} \quad (\text{A1.4b})$$

$$\begin{aligned}
r'_m &= \sqrt{x'^2 + y'^2 + z'^2} \\
\theta' &= \cos^{-1} \frac{z'}{r'_m} \\
\varphi' &= \tan^{-1} \frac{y'}{x'}
\end{aligned} \tag{A1.4c}$$

With equation (A1.4c) computed, all parameters on equation (A1.1) can be calculated. The vector transformation is obtained by using:

$$\begin{bmatrix} E_{Rm} \\ E_{\theta m} \\ E_{\varphi m} \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \varphi & \sin \theta \sin \varphi & \cos \theta \\ \cos \theta \cos \varphi & \cos \theta \sin \varphi & -\sin \theta \\ -\sin \varphi & \cos \varphi & 0 \end{bmatrix} \mathbf{I} \begin{bmatrix} \sin \theta' \cos \varphi' & \cos \theta' \cos \varphi' & -\sin \varphi' \\ \sin \theta' \sin \varphi' & \cos \theta' \sin \varphi' & \cos \varphi' \\ \cos \theta' & -\sin \theta' & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ E'_{\theta m} \\ E'_{\varphi m} \end{bmatrix} \tag{A1.5}$$

where \mathbf{I} is the identity matrix (3 x 3).

Equation (A1.5) transformed $\vec{E}_m(\vec{r}'_m)$ into $\vec{E}_m(\vec{r})$ in the reference coordinate system. This process is repeated for each array element. Notice that no constraints have been put into equation (A1.3) regarding the observation distance. This expression (eq. (A1.3)) is valid everywhere except at the location of the source itself. This formulation assumes that each pattern is boresighted in the +z direction. However, the identity matrix \mathbf{I} in equation (A1.5) could be replaced by a rotation matrix (Euler matrix of transformation) to account for arbitrary pointing.

The array radiation pattern is usually divided into two orthogonal polarizations. Equation (A1.3) may be written:

$$\vec{E}(\vec{r}) = \sum_{m=1}^M \vec{E}_m(\vec{r}) = \sum_{m=1}^M E_{\theta m}(\vec{r}) \hat{\theta} + \sum_{m=1}^M E_{\varphi m}(\vec{r}) \hat{\varphi} \tag{A1.5a}$$

which can be expressed as:

$$\vec{E}(\vec{r}) = E_{\theta} \hat{\theta} + E_{\varphi} \hat{\varphi} \tag{A1.5b}$$

The orthogonal components described in equation (A1.5b) are the usual spherical components. Another way of dividing the electric field into two orthogonal polarization is by using Ludwig's definition 3 (ref. 12). The following polarization vectors are introduced:

$$\hat{R} = \hat{\theta} (ae^{j\psi} \cos \varphi + b \sin \varphi) + \hat{\varphi} (-ae^{-j\psi} \sin \varphi + b \cos \varphi) \quad (A1.5c)$$

$$\hat{C} = \hat{\theta} (ae^{-j\psi} \sin \varphi - b \cos \varphi) + \hat{\varphi} (ae^{-j\psi} \cos \varphi + b \sin \varphi) \quad (A1.5d)$$

The reference polarization and cross polarization expressions of $E(r)$ are:

$$\text{Reference polarization of } \vec{E}: E_R = \vec{E} \cdot \hat{R}^* \quad (A1.5e)$$

$$\text{Cross polarization of } \vec{E}: E_C = \vec{E} \cdot \hat{C}^* \quad (A1.5f)$$

With these expressions equation (A1.5a) can be rewritten as:

$$\vec{E}(\vec{r}) = E_R \hat{R} + E_C \hat{C} \quad (A1.5g)$$

The parameters a , b , and ψ can be obtained from table I.

POWER RADIATED

The total power radiated (time-averaged) by the array is given by:

$$P_{\text{rad}} = \oint_S \text{Re}(\vec{E} \times \vec{H}^*) \cdot d\vec{s} \quad (B.1)$$

where

$\nabla \times \vec{E} = -j\omega\mu_0\vec{H}$ Maxwell equation
 $ds = \hat{a}_r r^2 \sin \theta d\theta d\varphi$ differential surface area
 S a sphere of radius r

In the far-field of the array (usually taken at $2\Delta^2/\lambda$, Δ is maximum array dimension), the power radiated given in equation (B.1) can be simplified to:

$$P_{\text{rad}} = \oint_S \frac{\vec{E}(\vec{r}) \cdot \vec{E}^*(\vec{r})}{Z_0} r^2 \sin \theta d\theta d\varphi \quad (B.2)$$

(Z_0 is free space impedance)

Substituting equation (A1.3) into equation (B.2) gives:

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} \left(\sum_{m=1}^M \vec{E}_m(\vec{r}) \right) \cdot \left(\sum_{n=1}^M \vec{E}_n(\vec{r}) \right)^* \frac{r^2}{Z_0} \sin \theta d\theta d\varphi \quad (B.3)$$

In general, the above expression does not have a closed form solution and is evaluated numerically using a Romberg integration algorithm (ref. 13).

Using far-field approximations, equation (A1.1) can be simplified as follows:
(This will restrict the observation distance to be only in the far-field of the array.)

$$\vec{E}(\vec{r}) = \sum_{m=1}^M \vec{E}_m(\vec{r}) e^{jk\hat{u} \cdot \vec{r}_m} \quad (\text{B.4})$$

where

\hat{u} unit vector, $\sin \theta \cos \varphi \hat{x} + \sin \theta \sin \varphi \hat{y} + \cos \theta \hat{z}$
 \vec{r}_m position vector, $x_m \hat{x} + y_m \hat{y} + z_m \hat{z}$
 $e^{jk\hat{u} \cdot \vec{r}_m}$ array factor component
 $\vec{E}_m(\vec{r})$ in equation (B.4) is given by:

$$\vec{E}_m(\vec{r}) = I_m \frac{e^{-jkr}}{r} \left[\hat{\theta}' U_{Em}(\theta') \left(a_m e^{j\psi_m} \cos \varphi' + b_m \sin \varphi' \right) + \hat{\varphi}' U_{Hm}(\theta') \left(-a_m e^{j\psi_m} \cos \varphi' + b_m \sin \varphi' \right) \right] \quad (\text{B.5})$$

With these far-field approximations all position vectors \vec{r}_m are parallel, making $(\hat{\theta}', \hat{\varphi}')$ equal $(\hat{\theta}, \hat{\varphi})$. No further coordinate transformation is required. Substituting equation (B.4) into equation (B.2) produces:

$$P_{\text{rad}} = \sum_{m=1}^M \sum_{n=1}^M I_m I_n^* \left(\frac{1}{Z_0} \int_0^{2\pi} \int_0^{\pi/2} A(\theta, \varphi) e^{jk\hat{u} \cdot (\vec{r}_m - \vec{r}_n)} \sin \theta \, d\theta \, d\varphi \right) \quad (\text{B.6})$$

where

$$A(\theta, \varphi) = \left[\left(U_{Em} U_{En}^* a_m a_n e^{j(\psi_m - \psi_n)} + U_{Hm} U_{Hn}^* b_m b_n \right) \cos^2 \varphi + \left(U_{Em} U_{En}^* b_m b_n + U_{Hm} U_{Hn}^* a_m a_n e^{j(\psi_m - \psi_n)} \right) \sin^2 \varphi + \left(U_{Em} U_{En}^* - U_{Hm} U_{Hn}^* \right) \left(a_m b_n e^{j\psi_m} + a_n b_m e^{-j\psi_n} \right) \sin \varphi \cos \varphi \right]$$

By defining

$$R_{mm} = \frac{1}{Z_0} \int_0^{2\pi} \int_0^{\pi/2} A(\theta, \varphi) e^{jk\hat{u} \cdot (\vec{r}_m - \vec{r}_n)} \sin \theta \, d\theta \, d\varphi \quad (\text{B.7})$$

the equation (B.6) can be expressed in the matrix form:

$$P_{\text{rad}} = \sum_{m=1}^M \sum_{n=1}^M R_{mn} I_m I_n^* \quad (\text{B.8})$$

The coefficient R_{mn} in equation (B.7) is a $M \times M$ matrix. The evaluation of R_{mn} is time consuming and it takes the most computer time in the analysis. Reference 10 shows a closed form solution to equation (B.7) for special case of the array element located in the x - y plane having identical polarization parameters.

DIRECTIVITY

The directivity is defined by

$$D(\theta, \varphi) = \frac{4\pi \frac{\vec{E}(\vec{r}) \cdot \vec{E}^*(\vec{r})}{Z_0} r^2}{P_{\text{rad}}} \quad (\text{C.1})$$

$D(\theta, \varphi)$ is known as total directivity. Also the reference directivity and the cross directivity can be easily obtained (ref. 14).

Reference directivity:

$$D_R(\theta, \varphi) = \frac{4\pi \frac{|\vec{E} \cdot \hat{R}|^2}{Z_0} r^2}{P_{\text{rad}}} \quad (\text{C.1a})$$

Cross directivity:

$$D_C(\theta, \varphi) = \frac{4\pi \frac{|\vec{E} \cdot \hat{C}|^2}{Z_0} r^2}{P_{\text{rad}}} \quad (\text{C.1b})$$

NUMERICAL RESULTS AND DISCUSSIONS

This section presents some numerical results to demonstrate the applications of the computer program. In order to substantiate the accuracy of the generalized array formulation and computer program, detailed comparisons were made with the results presented by King and Wong (ref. 2) and experimental data obtained at NASA Lewis (ref. 15). King and Wong reported on an $N \times N$ planar array configuration with symmetrical element patterns of the $\cos(\theta)^q$ -type. They used direct integration to compute the radiated power. The examples considered were an array of 2×2 elements for various element spacings and a 3×3 array for which element pattern and frequency were varied. In the NASA Lewis experimental case, a 2×2 array of rectangular

horns was used. In this case element pattern were measured in the array environment (to account for mutual coupling).

Very good agreement was obtained in all cases. The 2×2 array reported by King and Wong assumed a symmetrical E-H-plane patterns with a $q = 3.54$. Figure 4 shows a graphical description of the directivity as a function of element spacing for this array. The 3×3 array example used a symmetrical pattern but the performance is described as a function of element pattern, $(\cos(\theta))^q$, varying q and operating frequency. Table II shows the results from both approaches. In the NASA Lewis experimental case the 2×2 array was investigated relative to far-field patterns. These patterns were measured at three different scan angles (boresight, 3° and 5°). The element spacing and frequency were fixed ($S = 2.5 \lambda$, frequency = 30 GHz). These results are presented in figures 4(a) to (e).

A user guide for the programs developed is presented in appendix A. The implementation of equations (A1.3), (B.8), and (C.1) (antenna pattern, power, and directivity, respectively) with a computer program is given in appendix B. This program (appendix B) can be easily interfaced with available plotting routines for displaying the far-field antenna patterns. The numerical technique used to solve for equation (B.8) is not unique in any sense, but it was found to be faster than just using direct integration. Many other techniques can also be used, and easily implemented in the computer program (appendix B).

CONCLUDING REMARKS

One of the advantages of this generalized array formulation is that it does not break-down for special cases as might occur in approximations using closed forms. Also the formulation developed does not limit the pattern observation to the far-field zone. This can be very useful if the generalized formulation is going to be used with analysis programs for dual reflector configurations. The program developed can be easily modified to be implemented in the analysis of reflectors with phased array feeds.

This computer program is one of the key research tools at the NASA Lewis for analyzing advanced space communication antenna systems. The generalized formulation and computer program provides complete flexibility in analysis of array configurations and in the accurate analysis of experimental data.

APPENDIX A

USER GUIDE

PACAL1

Program Description

Given an angle ϕ and an array of sources, each with its current magnitude and relative phase, this program calculates the cross polarization, reference polarization, and the far field magnitude for a series of angles θ .

Input (FT05)

$\bar{X}, \bar{Y}, \bar{Z}$: coordinates of each source (in meters).
AMPL: current amplitude for each source (in amperes).
APHA: relative phase for each source (in radians).
M: total number of sources.
POL: denotes polarization
 1: linear polarization (x-direction)
 2: linear polarization (y-direction)
 3: right-hand circular polarization
 4: left-hand circular polarization
QE, QH: exponent of the cosine function that is used to approximate the element pattern in the analytic form.
INZT: interval between each elevation angle θ .
I: $1+\phi$, parameter to set for desired cut.

Extraneous Input: (to be changed accordingly)

line 2000 $\Delta WAVE = 3E8 / (\text{frequency})$
line 2500 $ZETA = (-(\text{total range of angles}) + (j-1) * INZT) * \pi / 180$

Output:

(FT08)

AZETA: value of θ at which the field values are taken (units degrees).
ERAD: far field magnitude.

(FT07)

AZETA: (see FT08)
AREF: reference polarization magnitude

(FT06)

AZETA: (see FT08)
ACR: cross polarization magnitude

Using the Program

Create an input file assigned to FT05 and output files assigned to FT06, FT07, and FT08. Running the program will fill FT06, FT07, and FT08 with data that can be used in the program ZPLOT1 to plot the appropriate graphs, while also printing the number of points calculated.

To run:

```
DDEF FT05F001,VS,IN1
DDEF FT06F001,VS,PAOUT3
DDEF FT07F001,VS,PAOUT2
DDEF FT08F001,VS,PAOUT1
PACAL1
```

Example of IN1:

```
&INPUT    X=.015,-.015,-.015,.015    Y=.015,.015,-.015,-.015
AMPL=1,1,1,1 APHA=0,0,0,0 M=4,QE=22,QH=16,INZT=.1,I=1 &END
```

PAA1

Program Description

This program takes a series of sources with different amplitudes and phases, and determines the power via two methods.

Input (FT09):

M: number of sources XX,YY,ZZ: x,y and z coordinates of source (meters) A: amplitude of source PHI: relative phase of source.

IPOL: denotex polarity

1: X-linear polarized

2: Y-linear polarized

3: circularly polarized

QE,QH: exponents of cosine functions..

Output:

(FT15) YPOWER: the power using direct integration method.
This serves as an input to PADTV.

PADTV

Program Description

This program requires the input file to PAA1 as well as the output file and then calculates the peak directivity.

Input:

(FT09) Input to PAA1

(FT15): Output to PAA1

Output (FT06)

DIR: peak directivity

ZPLOT1

Program Description

This program is designed to plot either the far-field, reference polarization, or cross polarization as calculated in PACAL1.

Input:

(FT06)

X(I),Y(J) (meters)

These values are the output of the previous program PACAL1. If the plot of the far field is desired, the file which was assigned to FT08 in PACAL1 should now be assigned to unit 6. Similarly, if the plot of the reference or cross polarization is desired, the file assigned to FT07 or FT06, respectively, should now be assigned to FT06.

Extraneous Input:

lines 1200,1250

IVARS=NP=total number of points-1

(This can be obtained from the output of PACAL1)

lines 3500,3600

VAR(4)=lower boundary of angles to be plotted.

VAR(5)=upper boundary of angles to be plotted.

Using the Program

After running PACAL1, there will be data in the files assigned to FT08, FT07, and FT06, which in this case will be denoted as A, B, and C. Assign either A, B, or C, to FT06 (after releasing (FT08, FT07, and FT06), depending on whether the far field, reference, or cross polarization plots are desired. Then, run the program with the appropriate plotting device (sidecar 4015) and the plot will appear.

To run:

```
RELEASE FT08
RELEASE FT07
RELEASE FT06
DDEF FT06F001,VS,PAOU1
ZPLOT1
```

If this routine is executed after that shown in the PACAL1 section, the far field will be plotted.

PROCDEF PAPLOT1

This procdef runs the programs PACAL1 and ZPLOT1 in succession while defining the necessary input and output devices.

As it is listed, file PA114 should contain the input to PACAL1. After assigning the devices FT08, FT07, and FT06 to files PAOUT1, PAOUT2, and PAOUT3, respectively, running PACAL1 will fill the respective file with the far field, reference polarization, and cross polarization magnitudes. After running PACAL1, setting the device FT06 to PAOUT1 will cause ZPLOT1 to plot the far field magnitude.

The Procdef:

```
ERASE PAOUT1 ERASE PAOUT2 ERASE PAOUT3 DDEF
FT08F001,VS,PAOUT1 DDEF FT07F001,VS,PAOUT2 DDEF
FT06F001,VS,PAOUT3 DDEF FT05F001,VS,PA114 PACAL1
RELEASE FT08 RELEASE FT07 RELEASE FT06 RELEASE FT05
GRAPH2D DDEF FT06F001,VS,PAOUT1 ZPLOT1
```

APPENDIX B

PCAL1

```

0000100 C*****THIS PROGARM WILL CALCULATE THE TRANVERSE ELECTRIC FIELD*****
0000200 C*****AND THE REFERENCE/CROSS POLARIZATION COMPONENTS OF THE FIELD*****
0000300
0000400
0000500
0000600 C*****DIMENSION STAMENTS*****
0000700
0000800     REAL INZT
0000900     DIMENSION X(100),Y(100),AMPL(100),APHA(100)
0001000
0001100
0001200 C*****INPUT DATA: X(I) ;X-COORDINATE OF HORN I,Y(I) ;Y-COORDINATE OF****
0001300 C*****HORN I.AMPL(I) ; AMPLITUDE OF I HORN, APHA(I) ; PHASE OF THE I ***
0001400 C*****HORN,M ; NUMBER OF HORNS,POL ; 1=X-POL,2=Y-POL,3-RHCP,4=LHCP.****
0001500 C*****QH ; EXPONENT H PLANE, QE ;EXPONENT OG E-PLANE*****
0001600
0001700
0001800     NAMELIST/INPUT/X,Y,Z,AMPL,APHA,M,POL,QH,QE,INZT,I
0001900     READ(5,INPUT)
0002000     INPH=1
0002100     J=0
0002200     AWAVE=3E8/10E9
0002300     PI=4.*ATAN(1.)
0002400     AK0=2.*PI/AWAVE
0002500 C*****SET THE ANGLES FOR THE PLOT*****
0002600
0002700 310 CONTINUE
0002800     J=J+1
0002900     ZETA=(-88.+(J-1)*INZT)*PI/180.
0003000     APhi=(I-1)*INPH*PI/180.
0003100
0003200
0003300     AZETA=ZETA*180./PI
0003400     AAPHI=APHI*180./PI
0003500     IF(AZETA.GT.88.)GO TO 11
0003600     IF(AAPHI.GT.180.)GO TO 999
0003700
0003800     RAZP=0.
0003900     AIMZP=0.
0004000     DO 20 K=1,M
0004100 C*****START THE ARRAY FACTOR SUMATION*****
0004200     ANG=APHA(K)+X(K)*SIN(ZETA)*COS(APHI)+Y(K)*SIN(ZETA)*SIN(APHI)
0004300     ANG=AK0*ANG
0004400     RAZP=RAZP+AMPL(K)*COS(ANG)
0004500     AIMZP=AIMZP+AMPL(K)*SIN(ANG)
0004600 20 CONTINUE
0004700 C*****SET POLARIZATION PARAMETERS*****
0004800     IF(POL.EQ.1.)GO TO 500
0004900     IF(POL.EQ.2.)GO TO 550
0005000     IF(POL.EQ.3.)GO TO 600
0005100     IF(POL.EQ.4.) GO TO 650
0005200     GO TO 999
0005300 500 A1=1.
0005400     A2=0.
0005500     SI=0.
0005600     GO TO 800
0005700 550 A1=0.
0005800     A2=1.
0005900     SI=0.
0006000     GO TO 800
0006100 600 A1=1./SQRT(2.)
0006200     A2=1./SQRT(2.)
0006300     SI=PI/2.
0006400     GO TO 800
0006500 650 A1=1./SQRT(2.)
0006600     A2=1./SQRT(2.)
0006700     SI=-PI/2.

```

```

0006800 800 CONTINUE
0006900
0007000
0007100 C*****START THE ELEMENT PATTERN CALCULATION*****
0007200 AUE=(COS(ZETA))*QE
0007300 AUH=(COS(ZETA))*QH
0007400 IF(AUE.LT.1E-3)AUE=1E-3
0007500 IF(AUH.LT.1E-3)AUH=1E-3
0007600 REEZL=A1*COS(SI)*COS(APHI)+A2*SIN(APHI)
0007700 REEZL=AUE*REEZL
0007800 AIMEZL=A1*SIN(SI)*COS(APHI)
0007900 AIMEZL=AUE*AIMEZL
0008000 REEPH=A2*COS(APHI)-A1*SIN(APHI)*COS(SI)
0008100 REEPH=AUH*REEPH
0008200 AIMEPH=-1*A1*SIN(APHI)*SIN(SI)
0008300 AIMEPH=AUH*AIMEPH
0008400
0008500
0008600
0008700 C*****START THE TOTAL FIELD CALCULATION AT ZETA PHI OBSERVATION*****
0008800
0008900 AREZE=RAZP*REEZL-AIMZP*AIMEZL
0009000 AIMZE=AIMZP*REEZL+RAZP*AIMEZL
0009100 AREPH=RAZP*REEPH-AIMZP*AIMEPH
0009200 AIMPH=RAZP*AIMEPH+AIMZP*REEPH
0009300 AEZ=AREZE*AREZE+AIMZE*AIMZE
0009400 AEP=AREPH*AREPH+AIMPH*AIMPH
0009500 ERAD=AEZ+AEP
0009600
0009700 C*****START THE CALCUIATION FOR CROSS POL AND THE REFERENCE POL*****
0009800 AB1=A1*COS(SI)*COS(APHI)+A2*SIN(APHI)
0009900 AB2=-1.*A1*SIN(SI)*COS(APHI)
0010000 AB3=-1.*A1*COS(SI)*SIN(APHI)+A2*COS(APHI)
0010100 AB4=A1*SIN(SI)*SIN(APHI)
0010200 AB5=A1*COS(SI)*SIN(APHI)-A2*COS(APHI)
0010300 AB6=A1*SIN(SI)*SIN(APHI)
0010400 AB7=A1*COS(SI)*COS(APHI)+A2*SIN(APHI)
0010500 AB8=A1*SIN(SI)*COS(APHI)
0010600
0010700
0010800
0010900 C*** START THE CROSS AND REFERENCE COMPUTATION*****
0011000 REER=(AREZE*AB1-AIMZE*AB2)+(AREPH*AB3-AIMPH*AB4)
0011100 AIMER=(AREZE*AB2+AIMZE*AB1)+(AREPH*AB4+AIMPH*AB3)
0011200 RECR=(AREZE*AB5-AIMZE*AB6)+(AREPH*AB7-AIMPH*AB8)
0011300 AIMCR=(AREZE*AB6+AIMZE*AB5)+(AREPH*AB8+AIMPH*AB7)
0011400 AREF=REER*REER+AIMER*AIMER
0011500 ACR=RECR*RECR+AIMCR*AIMCR
0011600 WRITE(8,400)AZETA,ERAD
0011700 WRITE(7,400)AZETA,AREF
0011800 WRITE(6,400)AZETA,ACR
0011900 400 FORMAT(5X,E15.5,5X,E15.5)
0012000
0012100
0012200 GO TO 310
0012300 11 WRITE(12,888)J
0012400 888 FORMAT(5X,'TOTAL NUMBER OF POINTS',I5)
0012500
0012600
0012700
0012800
0012900
0013000 999 STOP
0013100 END

```

PAA1

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0000100    DIMENSION XX(100),YY(100)
0000200    DIMENSION AMAT(100,100),YAMAT(100,100)
0000300    DIMENSION GGXH(100),GGXE(100)
0000400    DOUBLE PRECISION BE(100,100),BH(100,100),CE(100,100),CH(100,100)
0000500    DIMENSION CDE(100,100),CDH(100,100),PHI(100),A(100),THETA(100,100)
0000600    DIMENSION AR(100),AI(100)
0000700    DOUBLE PRECISION QE,QH,P(100,100),XK,XXE,XXH
0000800    DOUBLE PRECISION BESE(100,100),BESH(100,100)
0000900    DOUBLE PRECISION ARHO
0001000    REAL LAMDA
0001100    DOUBLE PRECISION YBE(100,100),YBH(100,100),YCE(100,100),YCH(100,100)
0001200    DOUBLE PRECISION ZA(100),ZB
0001250    DOUBLE PRECISION XTHETA,DTHETA
0001260    DOUBLE PRECISION AP1,AP2
0001270    DOUBLE PRECISION AF11,AF12
0001300    C
0001400    C
0001500    C*****THIS ARE THE INPUTS TO POWER CALCULATION*****
0001600    C
0001700    NAMELIST/INPUT/M,XX,YY,ZZ,A,PHI,IPOL,QE,QH
0001800    READ (9,INPUT)
0001900    FREQ=30E9
0002000    LAMDA=3.E8/FREQ
0002100    PI=4.*ATAN(1.)
0002200    XK=2*PI/LAMDA
0002300
0002400
0002500
0002600
0002700    C*****THIS SECTION WILL CALCULATE THE DISTANCE AND ANGLE M,N*****
0002800    DO 40 JA=1,M
0002900    DO 50 IA=1,M
0003000    IF((XX(IA).EQ.XX(JA)).AND.(YY(IA).EQ.YY(JA)))GO TO 55
0003100    AL1=(XX(IA)-XX(JA))*(XX(IA)-XX(JA))
0003200    AL2=(YY(IA)-YY(JA))*(YY(IA)-YY(JA))
0003300    AB1=SQRT(AL1+AL2)
0003400    AA1=(XX(IA)-XX(JA))/AB1
0003500    AA2=(YY(IA)-YY(JA))/AB1
0003600
0003700    THETA(IA,JA)=AA1*AA1-AA2*AA2
0003800    P(IA,JA)=AB1
0003900    GO TO 89
0004000    55    P(IA,JA)=0.
0004100    THETA(IA,JA)=1.
0004200    89    CONTINUE
0004300
0004400
0004500    50    CONTINUE
0004600    40    CONTINUE
0004700    C*****THIS SECTION WILL COMPUTE THE BESSEL AND GAMMA FUNCTIONS*****
0004800    XXE=QE+.5
0004900    XXH=QH+.5
0005000    CALL GMMMA(XXE,GXXE,IER)
0005100    CALL GMMMA(XXH,GXXH,IER)
0005110    AF11=XXE+1
0005120    AF12=XXH+1
0005130    CALL GMMMA(AF11,ARE1,IER)
0005140    CALL GMMMA(AF12,ARH1,IER)
0005150    RATE=GXXE/ARE1
0005160    RATH=GXXH/ARH1
0005200    DO 60 JB=1,M
0005300    DO 70 IB=1,M
0005400    ARHO=XK*P(IB,JB)
0005500
0005600
0005700    BESE(IB,JB)=BESJP(ARHO,XXE)
0005800    BESH(IB,JB)=BESJP(ARHO,XXH)
0005900    70    CONTINUE

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0006000 60  CONTINUE
0006100 C
0006200
0006300 C*****THIS SECTION WILL CALCULATE THE BE(I,J) AND BH(I,J) COEFFICIENTS**
0006400
0006500
0006600 DO 80 JH=1,M
0006700 DO 90 IH=1,M
0006800 DTHETA=PI/200
0007000 YBE(IH,JH)=0
0007100 YBH(IH,JH)=0
0007150 YCE(IH,JH)=0.0
0007155 YCH(IH,JH)=0.0
0007180 IF(IH.EQ.JH)GO TO 91
0007200 DO 800 IL=1,100
0007300 XTHETA=DTHETA*(IL-1)
0007400 ZA(IL)=ABS(COS(XTHETA))
0007500 ZB=XK*P(IH,JH)*SIN(XTHETA)
0007600 IF(COS(XTHETA).LT.1E-10)GO TO 800
0007610 IF (ZB .LT. 1E-10) GO TO 806
0007650 FACT1=ZA(IL)**(2*QE)
0007655 FACT2=ZA(IL)**(2*QH)
0007659 AP1=0.
0007660 AP2=2.
0007680 ABE1=BESJP(ZB,AP1)
0007681 ABE2=BESJP(ZB,AP2)
0007700 YBE(IH,JH)=YBE(IH,JH)+FACT1*ABE1*SIN(XTHETA)*DTHETA
0008000 YBH(IH,JH)=YBH(IH,JH)+FACT2*ABE1*SIN(XTHETA)*DTHETA
0008300 YCE(IH,JH)=YCE(IH,JH)+FACT1*ABE2*SIN(XTHETA)*DTHETA
0008600 YCH(IH,JH)=YCH(IH,JH)+FACT2*ABE2*SIN(XTHETA)*DTHETA
0008900 GO TO 800
0009315 806 YBE(IH,JH)=YBE(IH,JH)+FACT1*SIN(XTHETA)*DTHETA
0009320 YBH(IH,JH)=YBH(IH,JH)+FACT2*SIN(XTHETA)*DTHETA
0009400 800 CONTINUE
0009500 IF (IH .EQ. JH) GO TO 91
0009600 BB1=(2**((QE-.5))*GXXE/((XK*P(IH,JH))*XXE)
0009700 BB2=(2**((QH-.5))*GXXH/((XK*P(IH,JH))*XXH)
0009800 BE(IH,JH)=BB1*BESE(IH,JH)
0009900 BH(IH,JH)=BB2*BESH(IH,JH)
0010000 GO TO 90
0010100 91 BE(IH,JH)=.5/XXE
0010200 BH(IH,JH)=.5/XXH
0010250 YBE(IH,JH)=.5*RATE
0010255 YBH(IH,JH)=.5*RATH
0010300
0010400
0010500 90 CONTINUE
0010600 80 CONTINUE
0010700 C
0010800
0010900 C*****THIS SECTION WILL CALCULATE THE CH(I,J) AND CE(I,J) COEFFICIENTS**
0011000 C
0011100 DO 100 JC=1,M
0011200 DO 110 IC=1,M
0011300 CE(IC,JC)=0
0011400 CH(IC,JC)=0
0011500 DO 120 KK=1,12
0011600 K=KK-1
0011700 FACT=1
0011800 DO 130 L=1,KK
0011900 130 FACT=FACT*L
0012000 XE=FLOAT(K)+QE+1.5
0012100 XH=FLOAT(K)+QH+1.5
0012200 CALL GMMMA(XE,GXE,IER)
0012300 CALL GMMMA(XH,GXH,IER)
0012400 IF (P(IC,JC) .EQ. 0) GO TO 225
0012500 CC1=GXXE/((4**K)*FACT*GXE)
0012600 CC2=(XK*P(IC,JC))* (2*K)
0012700 CDE(IC,JC)=CC1*CC2
0012800 AK=FLOAT(K)
0012900 IF (AMOD(AK,2.) .NE. 0) CDE(IC,JC)=-CDE(IC,JC)
0013000 GO TO 226

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0013100 225 CDE(IC,JC)=0
0013200 226 CE(IC,JC)=CE(IC,JC)+.5*CDE(IC,JC)
0013300 IF (P(IC,JC) .EQ. 0) GO TO 235
0013400 DD1=GXXH/((4**K)*FACT*GXH)
0013500 DD2=(XK*P(IC,JC))*((2**K)
0013600 CDH(IC,JC)=DD1*DD2
0013700 IF (AMOD(AK,2.) .NE. 0) CDH(IC,JC)=-CDH(IC,JC)
0013800 GO TO 236
0013900 235 CDH(IC,JC)=0
0014000 236 CH(IC,JC)=CH(IC,JC)+.5*CDH(IC,JC)
0014100 120 CONTINUE
0014200 CH(IC,JC)=CH(IC,JC)-BH(IC,JC)
0014300 CE(IC,JC)=CE(IC,JC)-BE(IC,JC)
0014400 110 CONTINUE
0014500 100 CONTINUE
0014600 C
0014700 C
0014800 C CALCULATING THE POWER
0014900 C
0015800 ZREAL=0.
0015900 ZIMAG=0.
0015940 YZREAL=0
0015950 YZIMAG=0
0016000 DO 310 ID=1,M
0016100 AR(ID)=A(ID)*COS(PHI(ID))
0016200 310 AI(ID)=A(ID)*SIN(PHI(ID))
0016300 IF(IPOL.EQ.1)GO TO 150
0016400 IF(IPOL.EQ.2)GO TO 151
0016500 GO TO 152
0016600 150 DO 320 J=1,M
0016700 DO 330 I=1,M
0016800 AMAT(I,J)=BE(I,J)+BH(I,J)+THETA(I,J)*(CH(I,J)-CE(I,J))
0016850 YAMAT(I,J)=YBE(I,J)+YBH(I,J)+THETA(I,J)*(YCH(I,J)-YCE(I,J))
0016900 330 CONTINUE
0017000 320 CONTINUE
0017100 GO TO 160
0017200 151 DO 220 J=1,M
0017300 DO 230 I=1,M
0017400 AMAT(I,J)=BE(I,J)+BH(I,J)+THETA(I,J)*(CE(I,J)-CH(I,J))
0017450 YAMAT(I,J)=YBE(I,J)+YBH(I,J)+THETA(I,J)*(YCE(I,J)-YCH(I,J))
0017500 230 CONTINUE
0017600 220 CONTINUE
0017700 GO TO 160
0017800 152 DO 420 J=1,M
0017900 DO 430 I=1,M
0018000 AMAT(I,J)=BE(I,J)+BH(I,J)
0018050 YAMAT(I,J)=YBE(I,J)+YBH(I,J)
0018100 430 CONTINUE
0018200 420 CONTINUE
0018300 160 CONTINUE
0018400 DO 520 J=1,M
0018500 DO 530 I=1,M
0018600 PREAL=(AR(I)*AR(J)+AI(I)*AI(J))*AMAT(I,J)
0018620 YPREAL=(AR(I)*AR(J)+AI(I)*AI(J))*YAMAT(I,J)
0018650 PIMAG=(AR(J)*AI(I)-AR(I)*AI(J))*YAMAT(I,J)
0018700 PIMAG=(AR(J)*AI(I)-AR(I)*AI(J))*AMAT(I,J)
0018800 ZREAL=ZREAL+PREAL
0018900 ZIMAG=ZIMAG+PIMAG
0018940 YZREAL=YZREAL+YPREAL
0018960 YZIMAG=YZIMAG+YFIMAG
0019000 530 CONTINUE
0019100 520 CONTINUE
0019800 ZREAL=(1/120.)*ZREAL
0019900 ZIMAG=(1/120.)*ZIMAG
0019950 YZREAL=(1/120.)*YZREAL
0019960 YZIMAG=(1/120.)*YZIMAG
0020000 POWER=SQRT(ZREAL**2+ZIMAG**2)
0020050 YPOWER=SQRT(YZREAL**2+YZIMAG**2)
0020100 C

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0020200 C POWER OF SINGLE ELEMENT
0020300 SINGLE=(QE+QH+1)/(60*(2*QE+1)*(2*QH+1))
0020400 RATIO=POWER/SINGLE
0020450 YRATIO=YPOWER/SINGLE
0020500
0020600
0020700 WRITE(6,500)SINGLE
0020800 500 FORMAT(5X,'POWER OF A SINGLE ELEMENT=',1X,E15.5)
0020900 WRITE(6,501)RATIO,YRATIO
0021000 501 FORMAT(///5X,'CLOSED FORM RATIO=',1X,E15.5,'DIRECT INT RATIO=',
1X,E15.5)
0021100 WRITE(6,502)POWER,YPOWER
0021200 502 FORMAT(///5X,'CLOSE FORM POWER=',1X,E15.5,'DIRECT INT POWER=',1X,
E15.5)
0021210 WRITE (15,600) YPOWER
0021220 WRITE (16,600) POWER
0021230 600 FORMAT (E15.5)
0021300 C*****SPECIAL CASES FOR CHECKING RESULTS PREVIOUSLY CALCULATED*****
0021400
0021500
0021600
0021700 C*****LARGE SPACING CASE 6 LAMDA OR GREATER*****
0021800
0021900 POWER1=0.
0022000 DO 550 I=1,M
0022100 POWER1=POWER1+A(I)
0022200 550 CONTINUE
0022300 POWER1=SINGLE*POWER1
0022350 IF(IPOL.GE.3) GO TO 810
0022355 GO TO 811
0022358 810 POWER1=2.*POWER1
0022359 811 CONTINUE
0022400 WRITE(6,503)POWER1
0022500 503 FORMAT(5X,'LARGE SPACING POWER=',1X,E15.5)
0022600
0022700
0022800 C*****SYMMETRIC PATTERN QH=QE POWER CALCULATION*****
0022900 RECF=0.
0023000 REDI=0.
0023100 RIMCF=0.
0023200 RIMDI=0.
0023300 DO 560 J=1,M
0023400 DO 561 I=1,M
0023500 CA22=(AR(I)*AR(J)+AI(I)*AI(J))*BE(I,J)
0023600 CA33=(AR(I)*AR(J)+AI(I)*AI(J))*YBE(I,J)
0023700 CA44=(AR(J)*AI(I)-AR(I)*AI(J))*BE(I,J)
0023800 CA55=(AR(J)*AI(I)-AR(I)*AI(J))*YBE(I,J)
0023900 RECF=RECF+CA22
0024000 REDI=REDI+CA33
0024100 RIMCF=RIMCF+CA44
0024200 RIMDI=RIMDI+CA55
0024300 561 CONTINUE
0024400 560 CONTINUE
0024500 POLPCF=SQRT(RECF**2+RIMCF**2)*(1/60.)
0024600 POLPDI=SQRT(REDI**2+RIMDI**2)*(1/60.)
0024650 IF(IPOL.GE.3)GO TO 710
0024750 GO TO 711
0024800 710 POLPCF=2.*POLPCF
0024900 POLPDI=2.*POLPDI
0024950 711 CONTINUE
0025000 WRITE(6,510)POLPCF,POLPDI
0025100 510 FORMAT(///5X,'POWER CLOSE FORM SYM=',1X,E15.5,'POWER D I SYM=',1X,
E15.5)
0025200 STOP
0025300 END

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PADTV

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0000100      DIMENSION THETA(100),PHI(100),ETHETA(100,100),XX(100)
0000200      DIMENSION YY(100),AMP(100),PHASE(100)
0000400      DIMENSION AR(100,100),AI(100,100),AA(100,100)
0000500      DIMENSION XRADI(100,100),EPHI(100,100)
0000550      DIMENSION UE(100),UH(100),XIL(100,100)
0000560      DIMENSION A(100)
0000600      C
0000700      C
0000800      C READING VARIABLES
0000900      C
0001000      NAMELIST/INPUT/M,XX,YY,ZZ,QE,QH,A,PHI,IPOL
0001100      READ (9,INPUT)
0001105      DO 130 I=1,M
0001110      AMP(I)=A(I)
0001115      PHASE(I)=PHI(I)
0001120      130 CONTINUE
0001125      READ (15,131) POWER
0001130      131 FORMAT (E15.5)
0001200      C
0001300      PI=4.*ATAN(1.)
0001400      FREQ=29.5E9
0001500      XLAMDA=3.E8/FREQ
0001600      XK=2*PI/XLAMDA
0001700      ZO=377
0001800      C
0001900      C
0002000      C TESTING POLARITY
0002100      C
0002200      IF (IPOL .NE. 1) GO TO 102
0002300      C=1
0002400      B=0
0002500      PSI=0
0002600      GO TO 104
0002700      C
0002800      102 IF (IPOL .NE. 2) GO TO 103
0002900      C=0
0003000      B=1
0003100      PSI=0
0003200      GO TO 104
0003300      C
0003400      103 C=1/SQRT(2.)
0003500      B=1/SQRT(2.)
0003600      IF (IPOL .NE. 3) GO TO 105
0003700      PSI=PI/2
0003800      GO TO 104
0003900      105 PSI=-PI/2
0004000      C
0004100      C
0004200      C SOLVING EQUATIONS
0004300      C
0004400      104 DO 100 I=1,100
0004500      DO 101 J=1,100
0004600      C
0004700      THETA(I)=(FLOAT(I-1)/99)*PI-PI/2
0004800      PHI(J)=(FLOAT(J-1)/99)*PI-PI/2
0004900      C
0005000      UE(I)=(COS(THETA(I)))*QE
0005100      UH(I)=(COS(THETA(I)))*QH
0005200      C
0005300      AA1=UE(I)*UE(I)
0005350      AA2=(C*C)*(COS(PHI(J))*COS(PHI(J)))
0005360      AA3=(B*B)*(SIN(PHI(J))*SIN(PHI(J)))
0005370      AA4=2.*C*B*(COS(PSI)*COS(PHI(J))*SIN(PHI(J)))
0005380      ETHETA(I,J)=AA1*(AA2+AA3+AA4)
0005400      BB1=UH(I)*UH(I)
0005450      BB2=C*C*(SIN(PHI(J))*SIN(PHI(J)))
0005460      BB3=(B*B)*(COS(PHI(J))*COS(PHI(J)))
0005470      BB4=AA4
0005480      EPHI(I,J)=BB1*(BB2+BB3-BB4)

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0005500
0006000 C
0006100 XIL(I,J)=ETHETA(I,J)+EPHI(I,J)
0006200 AR(I,J)=0
0006300 AI(I,J)=0
0006400 C
0006500 DO 110 K=1,M
0006600 CC1=XX*(XX(K)*SIN(THETA(I))*COS(PHI(J)))
0006610 CC2=XX*(YY(K)*SIN(THETA(I))*SIN(PHI(J)))
0006620 EEXP=CC1+CC2
0006800 DREAL=AMP(K)*COS(PHASE(K)+EEXP)
0006900 AIMAG=AMP(K)*SIN(PHASE(K)+EEXP)
0007000 AR(I,J)=AR(I,J)+DREAL
0007100 AI(I,J)=AI(I,J)+AIMAG
0007200 110 CONTINUE
0007300 C
0007350 AA(I,J)=AI(I,J)*AI(I,J)+AR(I,J)*AR(I,J)
0007400 XRADI(I,J)=AA(I,J)*XIL(I,J)
0007500 101 CONTINUE
0007600 100 CONTINUE
0007700 C
0007800 C
0007900 C SORTING
0008000 C
0008100 DO 120 IA=1,100
0008200 DO 121 JA=2,100
0008300 IF (XRADI(IA,1) .GE. XRADI(IA,JA)) GO TO 121
0008400 XA=XRADI(IA,1)
0008500 XRADI(IA,1)=XRADI(IA,JA+1)
0008600 XRADI(IA,JA+1)=XA
0008700 121 CONTINUE
0008800 120 CONTINUE
0008900 C
0009000 DO 122 IB=2,100
0009100 IF (XRADI(1,1) .GE. XRADI(IB,1)) GO TO 122
0009200 XB=XRADI(IB,1)
0009300 XRADI(1,1)=XRADI(1,1)
0009400 XRADI(1,1)=XB
0009500 122 CONTINUE
0009600 C
0009650 WRITE(8,973)XRADI(1,1)
0009651 973 FORMAT(5X,'THIS IS THE PEAK VALUE',3X,E15.5)
0009700 DIR=4.*PI*XRADI(1,1)/(POWER*ZO)
0009710 DIR =10.*ALOG10(DIR)
0009800 WRITE (6,123) DIR
0009900 123 FORMAT (5X,'DIRECTIVITY=',E15.5)
0010000 STOP
0010100 END

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ZPLOT1

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0000100 C*****THIS PROGRAM CAN BE USED TO PLOT THE ANTENNA
0000101 C*****FAR-FIELD PATTERN.(E-PLANE OR H-PLANE CUTS)
0000103 C*****BY R.J. ACOSTA
0000200     DIMENSION X(5000),Y(5000),IVARS(20),VARS(20),RTNARR(2)
0000300
0000400     DIMENSION XTITLE(5),YTITLE(5)
0000401     LOGICAL*1 IAXIS
0000402     INTEGER*2 N1
0000500     DATA XTITLE/'ELEV','ATIO','N AN','GLE ','DEG.'//
0000600     DATA YTITLE/'RELA','TIVE',' AMP','LITU','DE '//
0000700     CALL TITLE(4,20,15,XTITLE)
0000800     CALL TITLE(3,17,15,YTITLE)
0000900
0000901
0001000
0001001
0001100     IVARS(1)=2
0001200     IVARS(2)=1760
0001250     NP=1760
0001310
0001410     N1=NP
0001510
0001610
0001710
0001810
0001910
0002000     DO 15 J=1,NP
0002100     READ(6,300)X(J),Y(J)
0002200 300     FORMAT(5X,E15.5,5X,E15.5)
0002300 15     CONTINUE
0002400     CALL SCLBAK(.FALSE.,N1,Y,RTNARR)
0002410     VMAX=RTNARR(2)
0002420
0002500
0002600
0002700     DO 16 J=1,NP
0002705     Y(J)=Y(J)/VMAX
0002710     IF(Y(J).LT.1E-8)Y(J)=1E-8
0002720     Y(J)=10.*ALOG10(Y(J))
0002740 16     CONTINUE
0002800
0002900
0003000
0003100
0003200     VARS(1)=9.
0003300     VARS(2)=8.
0003400     VARS(3)=0.
0003500     VARS(4)=-90.
0003600     VARS(5)=90.
0003700     VARS(6)=6.
0003800     VARS(7)=2.
0003900     VARS(8)=-1.
0004000     VARS(9)=0.
0004100     CALL XAXIS(-1.,-1.,VARS)
0004200     VARS(2)=9.
0004300     VARS(3)=90.
0004400     VARS(5)=0.
0004500     VARS(6)=8.
0004600     VARS(4)=-80.
0004700     CALL YAXIS(-1.,-1.,VARS)
0004800     CALL GPLOT(X,Y,IVARS)
0004900     CALL DISPLA(1)
0005000
0005001     READ(9,993)XYZ
0005002 993     FORMAT(1A1)
0005004     CALL TERM
0005100     STOP
0005200     END

```

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TABLE I. - POLARIZATION PARAMETERS

Polarization type	a_m	b_m	ψ_m
Linear-X	1	0	0
Linear-Y	0	1	0
RHCP ^a	1/2	1/2	0.5 π
LHCP ^b	1/2	1/2	-0.5 π

^aRHCP (right-hand circular polarized).

^bLHCP (left-hand circular polarized).

TABLE II. - COMPARISON OF DIRECTIVITY RESULTS WITH THOSE OBTAINED BY KING AND WONG (ref. 2)

Frequency, MHz	S/λ	Element pattern HPBW, deg	Pattern, $\cos^2(\theta)$ $q_E = q_H$	King-Wong measured directivity, dB	King-Wong, dB calculated	NASA Lewis, dB calculated
450	0.687	86.0	1.11	17.1	17.3	17.10
500	.763	92.0	0.96	17.9	18.0	17.83
600	.916	89.4	1.02	18.8	18.5	18.45
700	1.068	94.0	0.91	18.0	17.4	17.69
800	1.220	94.0	0.91	17.4	16.9	17.25

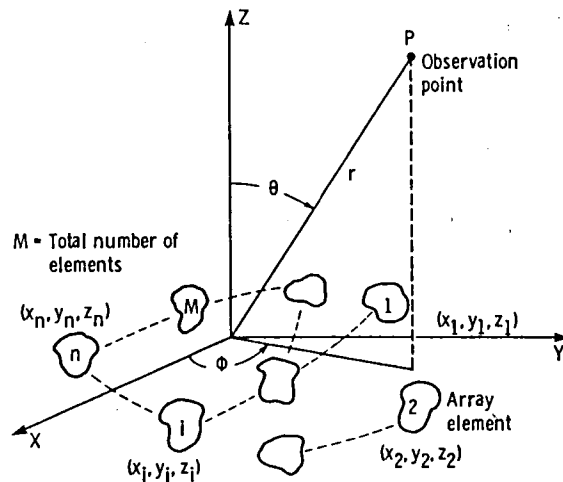


Figure 1. - Geometry of the generalized phased array.

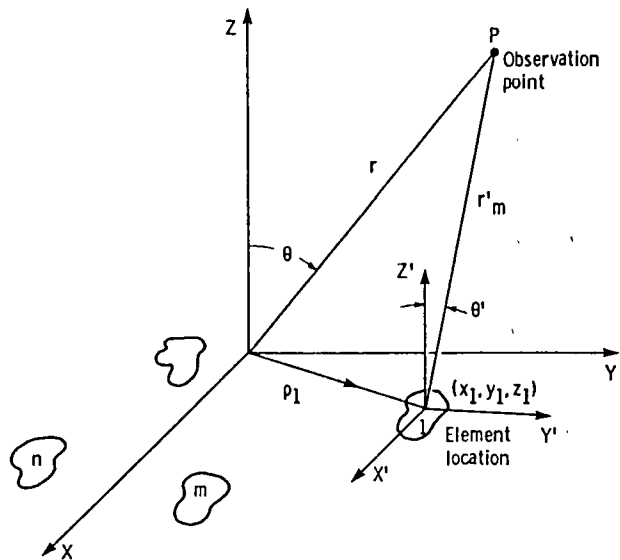


Figure 2. - A typical element coordinate system (X', Y', Z') and the reference coordinate system (X, Y, Z) .

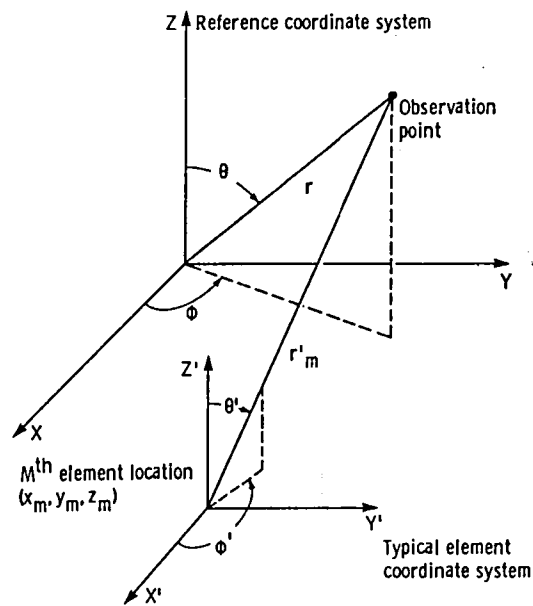


Figure 3. - Geometrical picture of the coordinate transformation between element coordinate system, and the reference coordinate system.

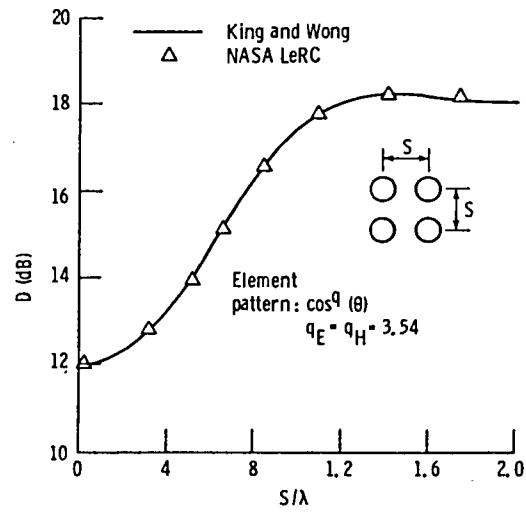
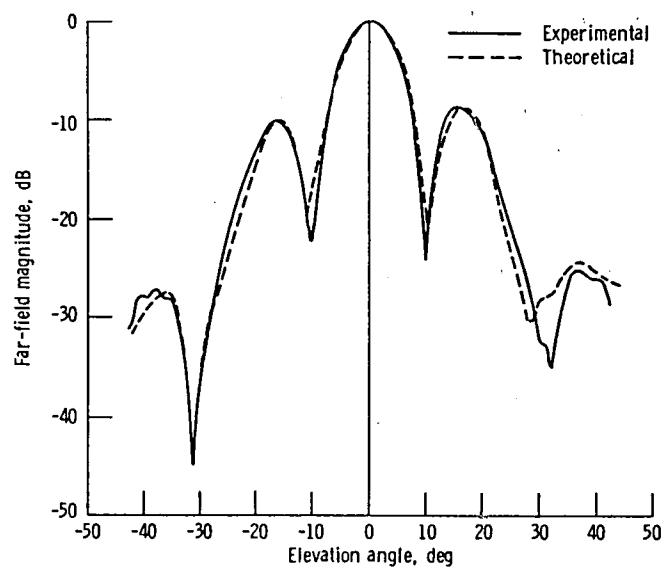
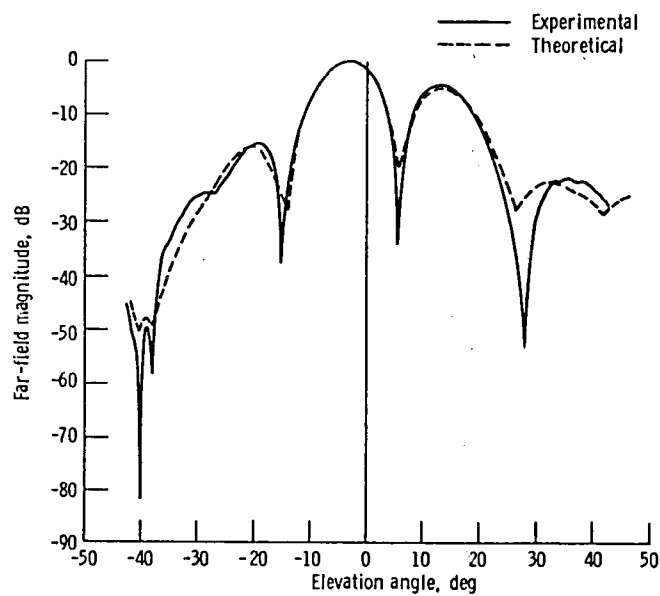


Figure 4. - Compared directivities for a 2x2 array as function of element spacing.



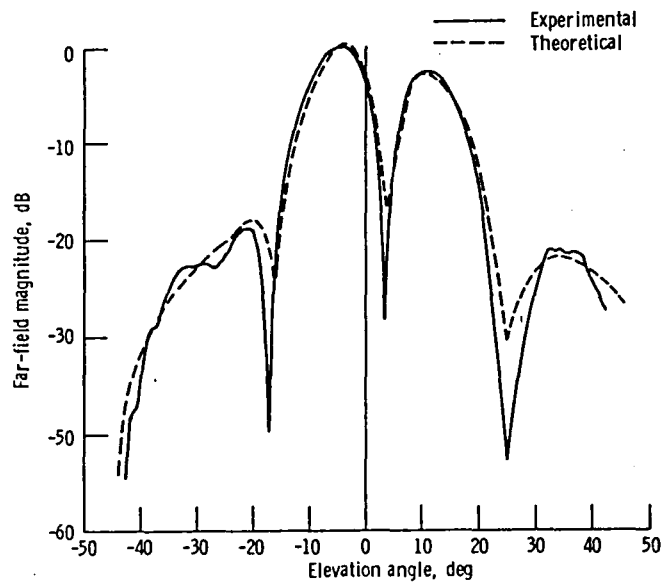
(a) Boresight, reference polarization far-field antenna pattern. (E-plane cut).

Figure 5.



(b) Scan case ($\theta_0 = 3.2^\circ$), reference polarization far-field antenna pattern. (E-plane cut).

Figure 5. - Continued.



(c) Scan case ($\theta_0 = 5.2^\circ$), reference polarization far-field antenna pattern (E-plane cut).

Figure 5. - Concluded.

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16. Abstract With the advent of monolithic microwave integrated circuit (MMIC) technology, the phased array has become a key component in the design of advanced antenna systems. Array-fed antennas are used extensively in today's multiple beam satellite antennas. In this report, a computer program based on a very efficient numerical technique for calculating the radiated power (Romberg integration), directivity and radiation pattern of a phased array is described. The formulation developed is very general, and takes into account arbitrary element polarization, E- and H-plane element pattern, element location, and complex element excitation. For comparison purposes sample cases have been presented. Excellent agreement has been obtained for all cases. Also included in appendixes A and B are a user guide and a copy of the computer program.					
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